بسم الله الرحمن الرحيم
Optical Electronics Course: Modulation Instability and Wavelength Conversion In Optical Fibers

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OUTLINE

1-Introduction, Applications and Motivation
2-Theory of Modulation Instability
3- Results and Discussions
1- Introduction

• Purpose: Access to new wavelength regions

• Variety of Applications:
  ✓ Agriculture
  ✓ Medical and Surgery
  ✓ Police
  ✓ Industry and …
Modulation Instability

- Modulation Instability (MI) is a general feature of wave propagation in dispersive nonlinear media and manifests itself in several branches of physics such as:
  - Nonlinear Optics
  - Fluid Dynamics
  - Plasma Physics
Scalar Modulation Instability (SMI)

- Scalar modulation instability, which leads to the breakup of an intense CW beam is the simplest form of MIs which can occur in optical fibers.
- It can be considered scalar in the sense that it is the only MI which can be described by the scalar single polarization nonlinear Schrodinger equation (NLSE).
Intensity, arb. un.

λ, nm
Applications

<table>
<thead>
<tr>
<th>FOPAs (Fiber-Optic Parametric Amplifiers)</th>
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<td>FOPOs (Fiber-Optic Parametric Oscillators)</td>
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Advantages compared with bulk OPAs / OPAs

- Wide Tunability
- Efficient Conversion
- Low Pump Power
Continuous-wave tunable optical parametric generation in a photonic-crystal fiber

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730-nm optical parametric conversion from near- to short-wave infrared band

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Widely tunable photonic crystal fiber Fabry–Perot optical parametric oscillator

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The traditional theoretical approach to the SMI analysis involves only terms up to $\beta_2$ in the Taylor series expansion.
Dispersion Regimes

- Normal: $\beta_2 > 0$
  SMI can be observed only if higher dispersion terms ($\beta_4$) are included.
- Anomalous: $\beta_2 < 0$
  SMI can be observed up to $\beta_2$. 

Zero-Dispersion Wavelength (ZDW)

Variation of $\beta_2$ with wavelength for bulk fused silica.
Photonic Crystal Fibers (PCFs)

- Lattice Pitch: $\Lambda$
- Air Hole Diameter: $d$
- Air Hole Shape
- Refractive Index of the Glass $n_{core}$
- Type of Lattice

The large waveguide contribution to the dispersion properties of PCFs $\beta_4$ is non-negligible
Nonlinear Schrodinger Equation describing the propagation of light in an optical fiber:

\[ i \frac{\partial A}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - i \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} + \frac{\beta_4}{24} \frac{\partial^4 A}{\partial T^4} + \gamma |A|^2 A = 0 \]

\( A \) is the slowly varying pulse envelope of the electric field.

\( T = t - \left( \frac{z}{v_g} \right) \) is the local time parameter.

\( \gamma = \frac{n_2 \omega_0}{c A_{\text{eff}}} \)

The nonlinear coefficient

\( A_{\text{eff}} \) is the effective area of the core
Steady state solution:

\[ A(z) = \sqrt{P_0} \exp(i \gamma P_0 z), \]

where \( P_0 \) is the incident power of the CW light at \( z = 0 \).

The stability of this CW solution to small perturbations can be examined by adding a small perturbation to the steady state solution such that

\[ A = [\sqrt{P_0} + a(z, T)]\exp(i \gamma P_0 z), \]

where \( a(z, T) \) is the perturbation and \(|a(z, T)| \ll P_0\).

\[
i \frac{\partial a}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 a}{\partial T^2} - i \frac{\beta_3}{6} \frac{\partial^3 a}{\partial T^3} + \frac{\beta_4}{24} \frac{\partial^4 a}{\partial T^4} + \gamma P_0 (a + a^*) = 0
\]
Assuming a general solution for the small perturbation of the form:

\[ a(z, T) = a_1 \exp[i(kz - \Omega t)] + a_2 \exp[-i(kz - \Omega t)] \]

where the coefficients \( a_1 \) and \( a_2 \) are real, \( k \) is the wave number and \( \Omega \) is the frequency of the perturbation.

\[
\begin{pmatrix}
  b_{11} & b_{12} \\
  b_{21} & b_{22}
\end{pmatrix}
\begin{pmatrix}
  a_1 \\
  a_2
\end{pmatrix} = 0,
\]

\[ b_{11} = -k + \frac{\beta_2}{2} \Omega^2 + \frac{1}{24} \beta_4 \Omega^4 + \gamma P_0. \]

\[ b_{12} = b_{21} = \gamma P_0 \]

\[ b_{22} = k + \frac{\beta_2}{2} \Omega^2 + \frac{1}{24} \beta_4 \Omega^4 + \gamma P_0. \]
A nontrivial solution exists only when the determinant of the matrix is zero:

\[ g(\Omega) = 2\ \text{Im}(k) \]

\[ = \pm \frac{1}{12} \text{Im}[(\sqrt{\beta_4 \Omega^2 + 12 \beta_2 \times \\
\sqrt{\beta_4 \Omega^4 + 12 \beta_2 \Omega^2 + 48 \gamma P_0}) \Omega] \]

The peak frequency shift of the SMI sidebands

\[ \Omega = \pm \sqrt{-2 \beta_4 (3 \beta_2 + \sqrt{9 \beta_2^2 - 6 \beta_4 \gamma P_0}) \frac{1}{\beta_4} } \]
Phase-Matching Analysis of Scalar Modulation Instability

\[ \Delta k = \Delta k_L + \Delta k_{NL} = 0, \]

\[ \Delta k_L = \beta(\omega_s) + \beta(\omega_a) - 2\beta(\omega_p). \]

\[ \beta(\omega_a) = \beta_p + \beta_1 \Omega + \frac{1}{2} \beta_2 \Omega^2 \]
\[ + \frac{1}{3!} \beta_3 \Omega^3 + \frac{1}{4!} \beta_4 \Omega^4. \]

\[ \beta(\omega_s) = \beta_p + \beta_1 (-\Omega) + \frac{1}{2} \beta_2 (-\Omega)^2 \]
\[ + \frac{1}{3!} \beta_3 (-\Omega)^3 + \frac{1}{4!} \beta_4 (-\Omega)^4. \]
\[ \Delta k_L = \beta_2 \Omega^2 + \frac{1}{12} \beta_4 \Omega^4. \]

\[ \Delta k_{NL} = \Delta k_{a,NL} + \Delta k_{s,NL} - 2\Delta k_{p,NL} \]

\[ = 2\gamma P_0 + 2\gamma P_0 - 2\gamma P_0 \]

\[ = 2\gamma P_0. \]

So, only the even-order terms of \( \beta \) contribute to SMI gain.
Phase Matching Analysis of SMI

\[ \Delta k = \Delta k_L + \Delta k_{NL} = 0 \]

\[ \Delta k_L = \beta (\omega_s) + \beta (\omega_a) - 2 \beta (\omega_p) \]

\[ \beta (\omega_a) = \beta_p + \beta_1 \Omega + \frac{1}{2} \beta_2 \Omega^2 + \frac{1}{3!} \beta_3 \Omega^3 + \frac{1}{4!} \beta_4 \Omega^4 \]

\[ \beta (\omega_s) = \beta_p + \beta_1 (-\Omega) + \frac{1}{2} \beta_2 (-\Omega)^2 + \frac{1}{3!} \beta_3 (-\Omega)^3 + \frac{1}{4!} \beta_4 (-\Omega)^4. \]

\[ \Delta k_L = \beta_2 \Omega^2 + \frac{1}{12} \beta_4 \Omega^4 \]

\[ \Delta k_{NL} = \Delta k_{a,NL} + \Delta k_{s,NL} - 2 \Delta k_{p,NL} = 2 \gamma P_0 + 2 \gamma P_0 - 2 \gamma P_0 = 2 \gamma P_0 \]
Normal vs. Anomalous Dispersion Regime

\[ \Delta k = \Delta k_L + \Delta k_{NL} = 0 \]

Anomalous Dispersion Regime:
\[ \Omega_{\text{max}} = \pm \sqrt{\frac{2\gamma P_0}{|\beta_2|}} \]

Normal Dispersion Regime:
\[ \Omega_{\text{max}} = \pm \sqrt{-2\beta_4 \left( 3\beta_2 + \sqrt{9\beta_2^2 - 6\beta_4\gamma P_0} \right)} / \beta_4 \]

Peak Frequency of SMI Sidebands
\[ g_{\text{max}} = 2\gamma P_0 \]
FWM Analysis of SMI

The propagation of CW light inside an optical fiber in the absence of loss is given by:

\[ i \frac{\partial A}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A = 0 \]

Total Transverse Electric Field Propagating along the Fiber

\[ E(x, y, z) = \begin{cases} 
  E_0(z) \exp(\imath k_0 z) \\
  + E_1(z) \exp(\imath k_1 z + \imath \Omega t) \\
  + E_2(z) \exp(\imath k_2 z - \imath \Omega t) \\
  + f(x, y) \exp(-\imath \omega_0 t) 
\end{cases} \]
Coupled Mode Equations

\[
\frac{dE_0}{dz} = i\gamma \left[ |E_0|^2 + 2 \left( |E_1|^2 + |E_2|^2 \right) \right] E_0 + 2\gamma E_1 E_2^* \exp(+i\Delta k_L z)
\]

\[
\frac{dE_1}{dz} = i\gamma \left[ |E_1|^2 + 2 \left( |E_2|^2 + |E_0|^2 \right) \right] E_1 + \gamma E_2^* E_0^2 \exp(-i\Delta k_L z)
\]

\[
\frac{dE_2}{dz} = i\gamma \left[ |E_2|^2 + 2 \left( |E_1|^2 + |E_0|^2 \right) \right] E_2 + \gamma E_1^* E_0^2 \exp(-i\Delta k_L z)
\]

\[E_j = A_j \exp\left(i \varphi_j\right) \; \text{for} \; j = 0, 1, 2.\]

Conservation of the total power among the waves:

\[P_0^* = A_0^2 + A_1^2 + A_2^2\]

Introducing normalized power for each wave:

\[
\eta(z) = \left[ A_0^2(z) \right] / P_0 \quad a_{1,2}(z) = A_{1,2} / P_0^{1/2} \quad \eta + a_1^2 + a_2^2 = 1
\]

Four Coupled Equations

\[ \frac{d\eta}{dz} = -4 \gamma P_0 \eta a_1 a_2 \sin \phi \]

\[ \frac{da_1}{dz} = \gamma P_0 \eta a_2 \sin \phi \]

\[ \frac{da_2}{dz} = \gamma P_0 \eta a_1 \sin \phi \]

\[ \frac{d\phi}{dz} = \Delta k_L + \gamma P_0 [2\eta - (a_1^2 + a_2^2)] + \gamma P_0 [\eta (\frac{a_1}{a_2} + \frac{a_2}{a_1}) - 4a_1 a_2] \cos \phi \]

\[ \phi(z) = \Delta k_L z + \phi_1(z) + \phi_2(z) - 2\phi_0(z) \]

Total phase difference of the process
3-Results and Discussions

Calculations have been done for a highly dispersive PCF:

$$\beta_2 = 2.5 \, ps^2 / km$$

$$\beta_4 = -4.49 \times 10^{-5} \, ps^4 / km$$

$$\gamma = 188 \, W^{-1} / km$$

$$P_0 = 10 \, W$$
SMI gain for both normal and anomalous dispersion regimes for $P_0 = 10W$
Power dependency of the peak frequency shifts in both dispersion regimes
With more details:

\( \beta_2 < 0 \)

\( \beta_2 > 0 \) & \( \beta_4 \)
H. Pakarzadeh, A. Keshavarz, and Z. Khadir, "Wavelength conversion based on four-wave mixing in photonic crystal fibers for achieving suitable wavelength region for skin applications," Proceedings of 19th Iranian Conference on Optics and Photonics, ICOP 2013, University of Sistan and Balouchestan, Zahedan (2013).
H. Pakarzadeh, A. Keshavarz, and Z. Khadir, "Wavelength conversion based on four-wave mixing in photonic crystal fibers for achieving suitable wavelength region for skin applications," Proceedings of 19th Iranian Conference on Optics and Photonics, ICOP 2013, University of Sistan and Balouchestan, Zahedan (2013).
H. Pakarzadeh, A. Keshavarz, and Z. Khadir, "Wavelength conversion based on four-wave mixing in photonic crystal fibers for achieving suitable wavelength region for skin applications," Proceedings of 19th Iranian Conference on Optics and Photonics, ICOP 2013, University of Sistan and Balouchestan, Zahedan (2013).
A low-amplitude, periodic perturbation can be amplified to form a train of high-intensity pulses through the modulation instability. The left panel shows how a small temporal modulation on a constant background (not visible on the scale used in the figure) evolves into a train of high power pulses at \( z=0 \) before decaying. In the frequency domain (right panel), this modulation is associated with multiple harmonic generation. Distance is plotted relative to a characteristic nonlinear length \( (L_{NL}) \). Time and frequencies are normalized relative to the fundamental modulation period \( (T_{mod}) \) and frequency \( (f_{mod}) \) respectively.

با تشکر از بذل توجه شما